Determine whether each table represents a *linear* or *nonlinear* function. Explain.

1.

x	−2	0	2	4
y	−1	0	1	2

SOLUTION:

Determine how the values change for x and y.

x	−2	0	2	4
y	−1	0	1	2

As x increases by 2 each time, y increases by 1 each time. The rate of change is constant, so this function is linear.

ANSWER:

Linear; rate of change is constant; as x increases by 2, y increases by 1.

2.

x	1	2	3	4
y	1	4	9	16

SOLUTION:

Determine how the values change for x and y.

x	1	2	3	4
y	1	4	9	16

Although x increases by 1 each time, y increases by a larger amount each time. The rate of change is not constant, so this function is nonlinear.

ANSWER:

Nonlinear; rate of change is not constant.

3.

x	5	10	15	20
y	13	28	43	58

SOLUTION:

Determine how the values change for x and y.

x	5	10	15	20
y	13	28	43	58

As x increases by 5 each time, y increases by 15 each time. The rate of change is constant, so this function is linear.

ANSWER:

Linear; rate of change is constant; as x increases by 5, y increases by 15.

4.

x	1	3	5	7
y	−2	−18	−50	−98

SOLUTION:

Determine how the values change for x and y.

x	1	3	5	7
y	−2	−18	−50	−98

Although x increases by 2 each time, y decreases by a larger amount each time. The rate of change is not constant, so this function is nonlinear.

ANSWER:

Nonlinear; rate of change is not constant.

5. The Guzman family drove from Anderson to Myrtle Beach . Use the table to determine whether the distance driven is a linear function of the hours traveled. Explain.

Time (h)	1	2	3	4
Distance (mi)	65	130	195	260

SOLUTION:

Determine how the values change for x and y.

Time (h)	1	2	3	4
Distance (mi)	65	130	195	260

The rate of change is constant. As the time increases by 1 hour, the distance increases by 65 miles, so this function is linear

ANSWER:

Yes; the rate of change is constant; as the time increases by 1 hour, the distance increases by 65 miles.

6. The table shows the height of several buildings in Chicago.

Building	Stories	Height (ft)
Harris Bank III	35	510
One Financial Place	40	515
Kluczynski Federal Building	45	545
Mid Continental Plaza	50	582
North Harbor Tower	55	552

Use the table to determine whether the height of the building is a linear function of the number of stories. Explain.

SOLUTION:

Determine how the values change for x and y.

Building	Stories	Height (ft)
Harris Bank III	35	510
One Financial Place	40	515
Kluczynski Federal Building	45	545
Mid Continental Plaza	50	582
North Harbor Tower	55	552

Although x increases by 5 each time, y changes by different amounts each time. The rate of change is not constant, so this function is nonlinear.

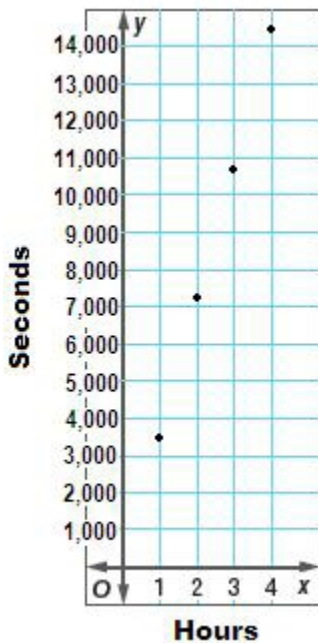*ANSWER:*

No; the rate of change is not constant.

7. There are 3,600 seconds in one hour. The total seconds is a function of the hours. Does this situation represent a linear or nonlinear function? Explain.

SOLUTION:

Make a table with two rows and five columns. Label the first row Hours(h) and the second row Seconds(s). Choose values for h and evaluate them in the expression $z = 3,600h$.

Hours (*h*)	1	2	3	4
Seconds (*s*)	3,600	7,200	10,800	14,400

Graph the points (h, s) on a coordinate plane. The ordered pairs (hours, seconds) lie on a straight line, so the situation represents a linear function.

ANSWER:

Linear; sample answer: If you graph the function, the ordered pairs (hours, seconds) lie on a straight line.

8. A football is placed on the ground to kick a field goal. The height of the ball is a function of the time in seconds. Does the path the football follows after being kicked represent a linear or nonlinear function? Explain.
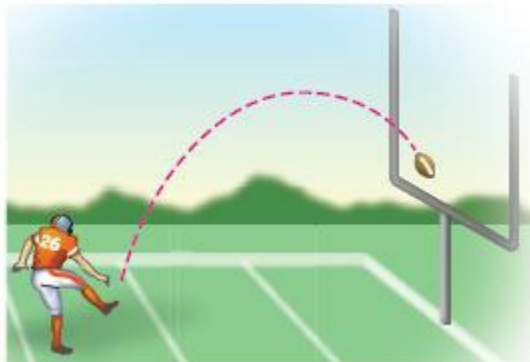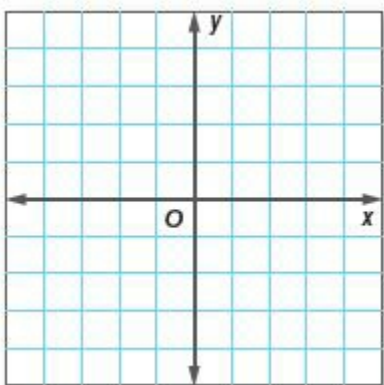
SOLUTION:

Nonlinear; sample answer: After being kicked, the ball will reach a maximum height and come back to the ground.

ANSWER:

Nonlinear; sample answer: After being kicked, the ball will reach a maximum height and come back to the ground.

Graph the function by making a table of ordered pairs. Determine whether the function is *linear* or *nonlinear*. Explain.
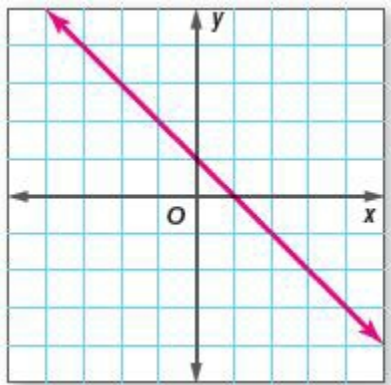
9. $y = -x + 1$

SOLUTION:

Select any four values for the domain x. Substitute these values for x to find the value of y. Graph each ordered pair. Draw a line that passes through each point.

Sample Table:

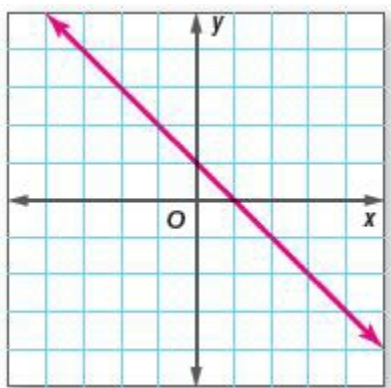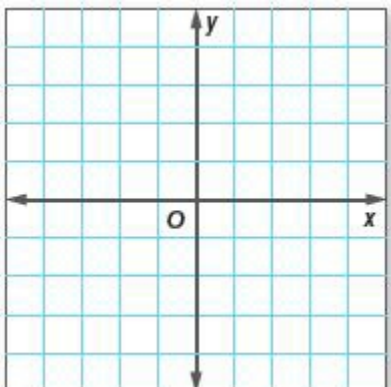| x	$-x + 1$	y	(x, y)
–2	–(–2) + 1	3	(–2, 3)
0	–(0) + 1	1	(0, 1)
1	–(1) + 1	0	(1, 0)
2	–(2) + 1	–1	(2, –1)

Graph:

Linear; sample answer: The points lie on a straight line.

ANSWER:

Linear; sample answer: The points lie on a straight line.

10. $y = \dfrac{-4}{x}$

SOLUTION:

Select any four values for the domain, x. Substitute these values for x to find the value of y. Graph each ordered pair. Draw a line that passes through each point.
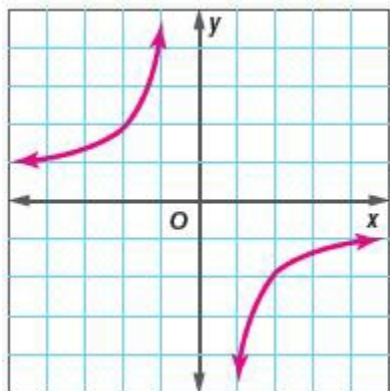
Sample Table:

x	$\dfrac{-4}{x}$	y	(x, y)
-3	$\dfrac{-4}{-3}$	$\dfrac{4}{3}$	$\left(-3, \dfrac{4}{3}\right)$

0	$\dfrac{-4}{0}$	Undefined	Undefined
3	$\dfrac{-4}{3}$	$\dfrac{-4}{3}$	$\left(3, \dfrac{-4}{3}\right)$
6	$\dfrac{-4}{6}$	$\dfrac{-2}{3}$	$\left(6, \dfrac{-2}{3}\right)$

Graph:
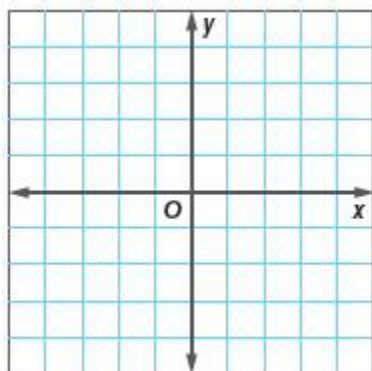

Nonlinear; sample answer: The graph is a curve.

ANSWER:
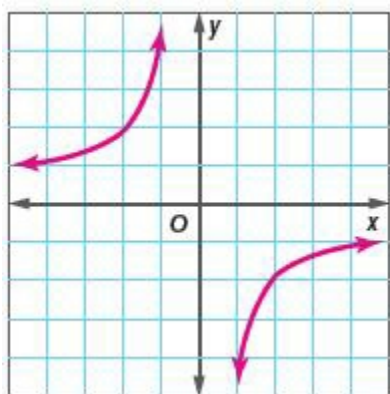
Nonlinear; sample answer: The graph is a curve.
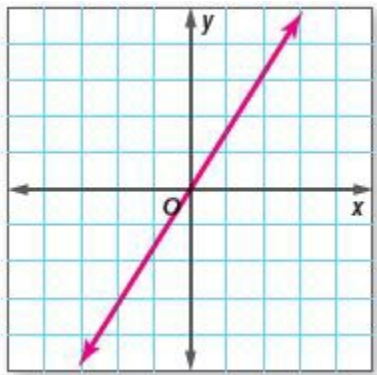
11. $y = \dfrac{3x}{2}$

SOLUTION:

Select any four values for the domain, x. Substitute these values for x to find the value of y. Graph each ordered pair. Draw a line that passes through each point.
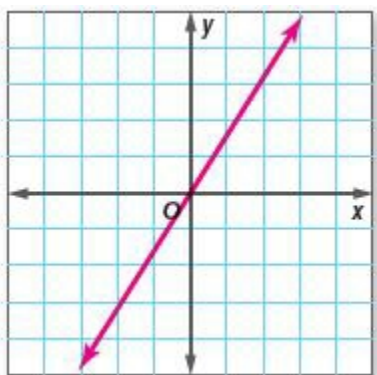
Sample Table:

x	$\dfrac{3x}{2}$	y	(x, y)
–4	$\dfrac{3(-4)}{2}$	–6	$(-4, -6)$
–2	$\dfrac{3(-2)}{2}$	–3	$(-2, -3)$
0	$\dfrac{3(0)}{2}$	0	$(0, 0)$
4	$\dfrac{3(4)}{2}$	6	$(4, 6)$

Graph:

Linear; sample answer: The points lie on a straight line.

ANSWER:

Linear; sample answer: The points lie on a straight line.

12. **Identify Structure** Complete the graphic organizer by determining if the graphs represent linear or nonlinear functions.

4-7 Linear and Nonlinear Functions

Linear or Nonlinear?
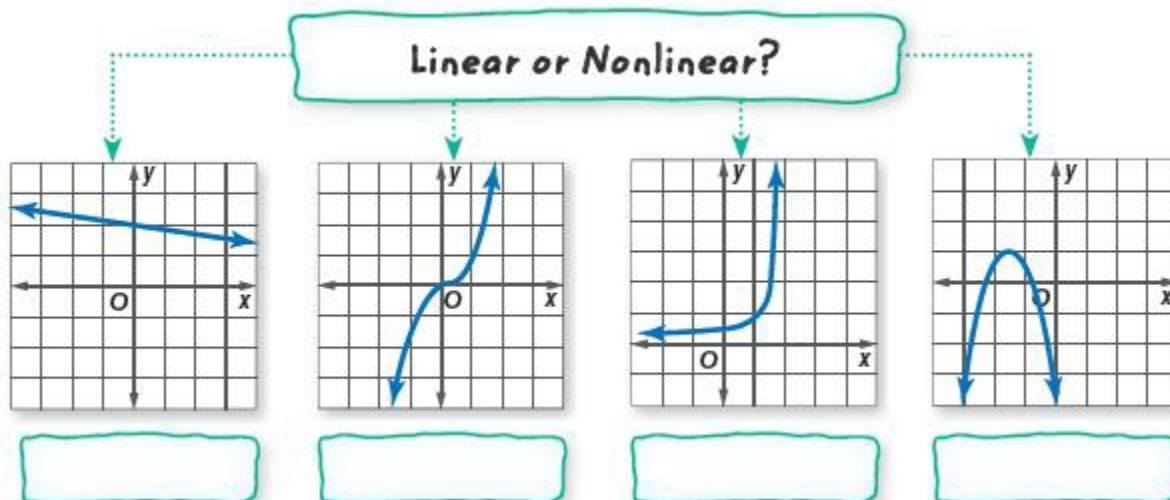
SOLUTION:

Linear functions have graphs that are straight lines. The first graph is the only graph that is a straight line. The rest are nonlinear.

Linear or Nonlinear?

Linear   Nonlinear   Nonlinear   Nonlinear

ANSWER:

Linear or Nonlinear?

Linear   Nonlinear   Nonlinear   Nonlinear
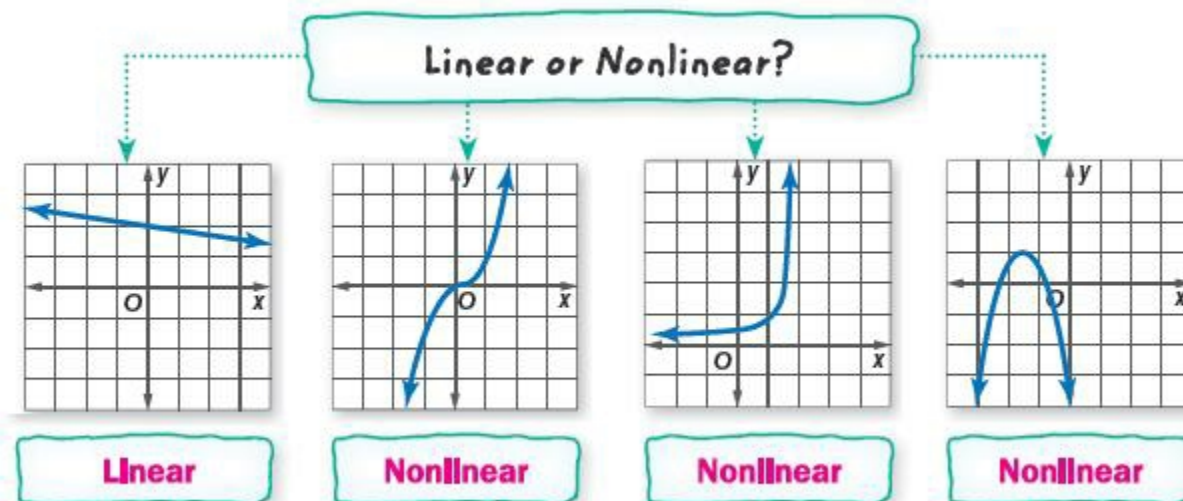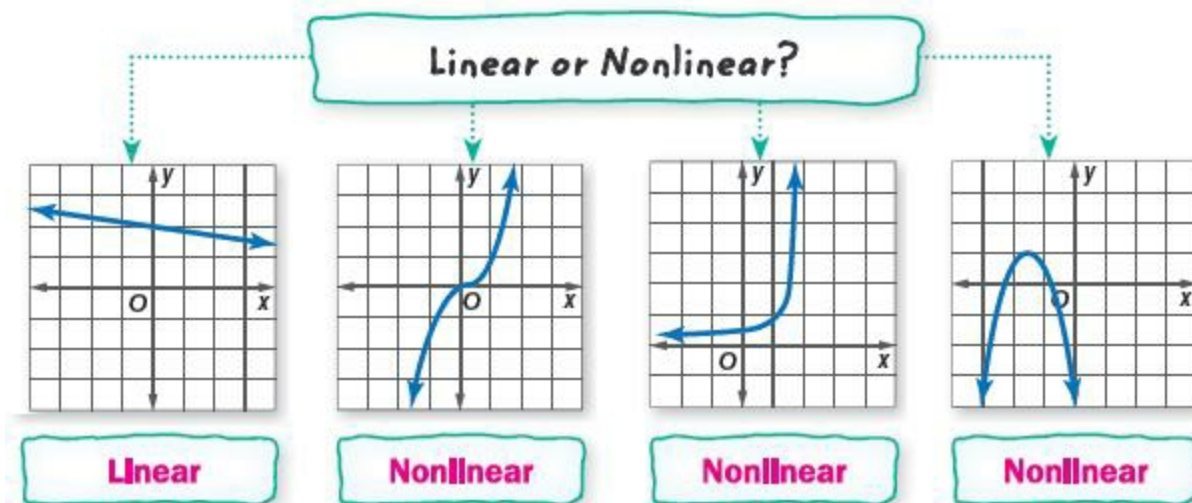
13. **Persevere with Problems** Does the graph shown represent a linear function? Explain.
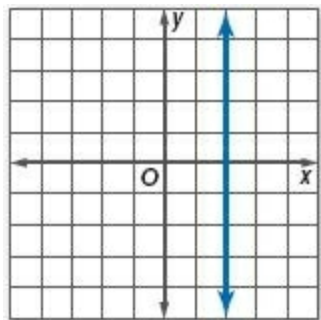
SOLUTION:

The graph is a line, however, the graphs of vertical lines are not functions because there is more than one value of y that corresponds to $x = 2$. So, the graph does not represent a linear function.

ANSWER:

No; sample answer: the graphs of vertical lines are not functions because there is more than one value of y that corresponds to $x = 2$.

14. **Model with Mathematics** Give an example of a situation that can be represented by a nonlinear function.

SOLUTION:

Sample answer: Every hour the number of bacteria in a petri dish doubles.

ANSWER:

Sample answer: Every hour the number of bacteria in a petri dish doubles.

15. **Reason Inductively** Explain how you can use different representations to determine whether a function is linear.

   SOLUTION:
   Sample answer: A non-vertical graph that is a straight line is linear. An equation that can be written in the form $y = mx + b$ is linear. If a table of values shows a constant rate of change, it is linear.

   ANSWER:
   Sample answer: A non-vertical graph that is a straight line is linear. An equation that can be written in the form $y = mx + b$ is linear. If a table of values shows a constant rate of change, it is linear.

   Determine whether each table represents a linear or nonlinear function. Explain.
16.

x	y
2	10
4	12
6	16
8	24

   SOLUTION:
   Determine how the values change for x and y.

x	y
2	10
4	12
6	16
8	24

   As x increases by 2, y increases by a greater amount each time. The rate of change is not constant, so this function is nonlinear.

   ANSWER:
   Nonlinear; rate of change is not constant.

17.

x	y
4	3
8	0
12	−3
16	−6

SOLUTION:

Determine how the values change for x and y.

x	y
4	3
8	0
12	−3
16	−6

As x increases by 4 each time, y decreases by 3 each time. The rate of change is constant, so this function is linear.

ANSWER:

Linear; rate of change is constant; as x increases by 4, y decreases by 3.

18. **Copy and Solve** The area of a square is a function of its perimeter. Graph the function on a separate sheet of grid paper. Explain whether the function is linear and if the graph is increasing or decreasing.

SOLUTION:

Create a table of values that has two rows and five columns. Label the rows for perimeter P and area A. Choose a side length for each square and then calculate the perimeter and area to complete the table. Use the formulas $P = 4s$ and $A = s^2$, where P is the perimeter of a square, A is the area of a square, and s is the side length of the square.

Perimeter *P* (units)	0	4	8	16
Area *A* (sq. units)	0	1	4	16

Graph the points as (P, A) on a coordinate plane.
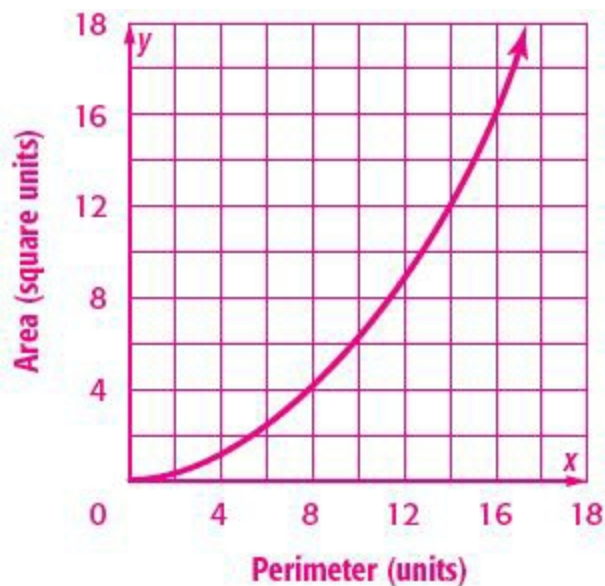
Sample answer: The function is nonlinear because the graph of the function is not a straight line and the line is increasing..

ANSWER:
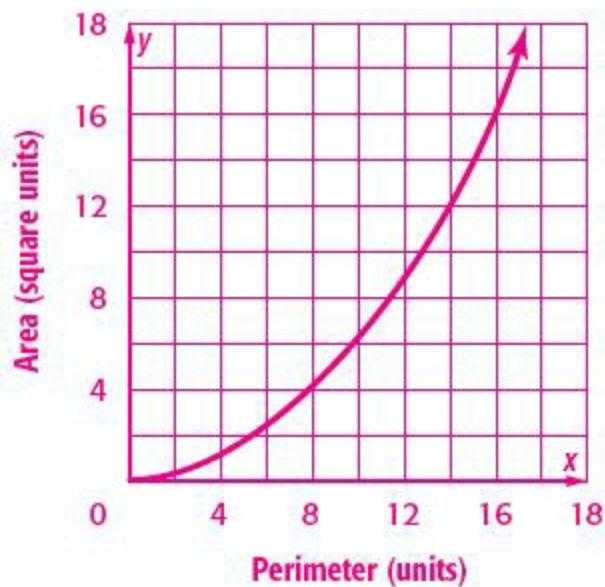
Sample answer: The function is nonlinear because the graph of the function is not a straight line and the line is increasing.

19. **Multiple Representations** Recall that the circumference of a circle is equal to two times π times its radius and that the area of a circle is equal to π times the square of the radius.

a. Tables Complete the table showing the circumference and area of circles with radius r.

4-7 Linear and Nonlinear Functions

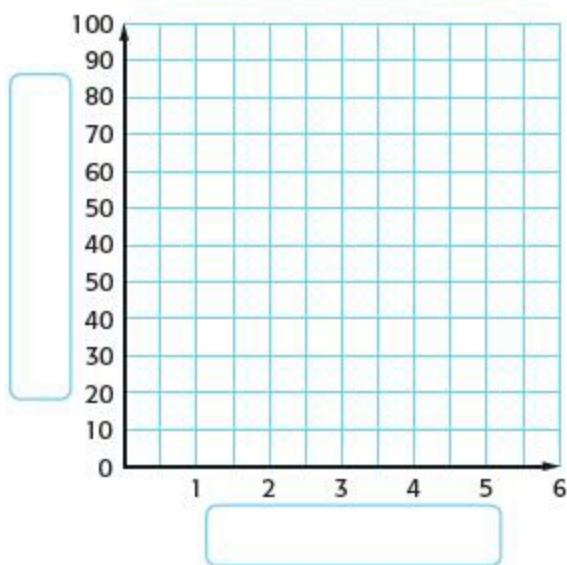| Radius r	Circumference 2 · π · r	Area πr²
1	$2 \cdot \pi \cdot 1 \approx 6.28$	$\pi \cdot 1^2 \approx 3.14$
2		
3		
4		
5		

b. Graphs Graph the ordered pairs (radius, circumference) and (radius, area) for each function on the same coordinate plane.

c. Words Is the circumference of a circle a linear or nonlinear function of its radius? the area? Explain your reasoning.

SOLUTION:

a. Substitute the values for the different radii into the expressions and simplify.

Radius r	Circumference 2 · π · r	Area πr²
1	$2 \cdot \pi \cdot 1 \approx 6.28$	$\pi \cdot 1^2 \approx 3.14$
2	$2 \cdot \pi \cdot 2 \approx 12.57$	$\pi \cdot 2^2 \approx 12.57$
3	$2 \cdot \pi \cdot 3 \approx 18.85$	$\pi \cdot 3^2 \approx 28.27$
4	$2 \cdot \pi \cdot 4 \approx 25.13$	$\pi \cdot 4^2 \approx 50.27$
5	$2 \cdot \pi \cdot 5 \approx 31.42$	$\pi \cdot 5^2 \approx 78.54$

b. Labe the *x*-axis as radius. Label the *y*-axis as circumference/area. Graph the ordered pairs (radius, circumference) then connect those points. Then graph the ordered pairs (radius, area) and connect those points.
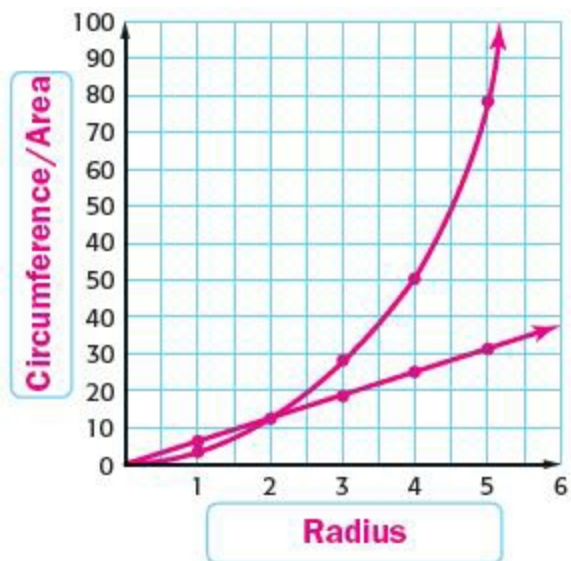
c. When you connect the points for the circumference, the graph is straight line so the function is linear. When you connect the points for area, they form a curve so the function is nonlinear.

ANSWER:

a.

Radius r	Circumference $2 \cdot \pi \cdot r$	Area πr^2
1	$2 \cdot \pi \cdot 1 \approx 6.28$	$\pi \cdot 1^2 \approx 3.14$
2	$2 \cdot \pi \cdot 2 \approx 12.57$	$\pi \cdot 2^2 \approx 12.57$
3	$2 \cdot \pi \cdot 3 \approx 18.85$	$\pi \cdot 3^2 \approx 28.27$
4	$2 \cdot \pi \cdot 4 \approx 25.13$	$\pi \cdot 4^2 \approx 50.27$
5	$2 \cdot \pi \cdot 5 \approx 31.42$	$\pi \cdot 5^2 \approx 78.54$

b.
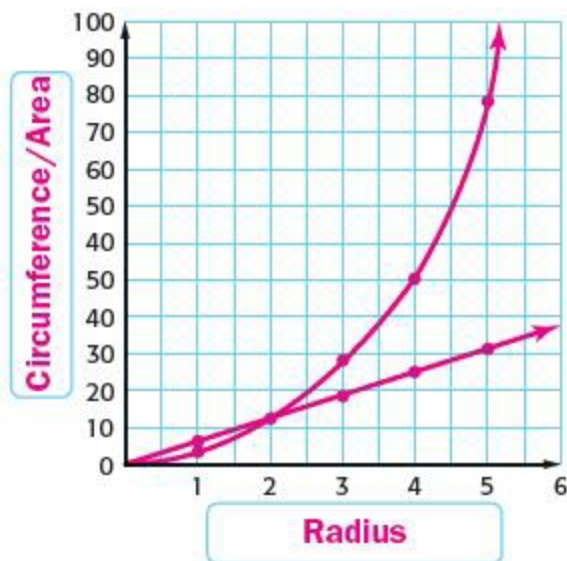
c. Circumference: linear; sample answer: When the ordered pairs are graphed, the points fall in a line. Area: nonlinear; sample answer: When the ordered pairs are graphed, the points do not fall in a line.

20. Determine if each table represents a linear or nonlinear function.

linear

nonlinear

x	y
9	1.25
11.5	2
14	2.75
16.5	3.5

x	y
−3	−6
−7	−1
−11	4
−15	9

x	y
10	5
13	7
16	10
19	14

x	y
2	15
4	20
6	25
8	30

SOLUTION:

x	y
9	1.25
11.5	2
14	2.75
16.5	3.5

This table represents a linear function because the *x*-values increase by 2.5 each time and the *y*-values increase by 0.75 each time.

x	y
−3	−6
−7	−1
−11	4
−15	9

This table represents a linear function because the *x*-values decrease by 4 each time and the *y*-values increase by 5 each time.

x	y
10	5
13	7
16	10
19	14

This table represents a nonlinear function because the *x*-values increase at a constant rate of 3, but the *y*-values increase by a greater amount each time.

x	y
2	15
4	20
6	25
8	30

This table represents a linear function because the *x*-values increase by 2 each time and the *y*-values increase by 5 each time.

ANSWER:

x	y
9	1.25
11.5	2
14	2.75
16.5	3.5

linear

x	y
−3	−6
−7	−1
−11	4
−15	9

linear

x	y
10	5
13	7
16	10
19	14

nonlinear

x	y
2	15
4	20
6	25
8	30

linear

21. Jung has $200 in a safe. Each month, he adds another $10 to the safe. Miguel opens a savings account with a $200 deposit and earns 2.5% interest each month on the total amount of money in the bank. Determine if each statement is

true or false.

a. The function representing Jung's savings is nonlinear.

b. The function representing Miguel's savings is linear.

c. After 1 year, Jung will have saved $320.

SOLUTION:

Create a table with three columns, one to represent the number of months, m, one for Jung's total savings, and one for Miguel's total savings. Jung starts with $200 and increases his savings by $10 a month. Miguel starts with $200 but he earns 10% interest on his money each month.

Month	Jung	Miguel
0	200	200
1	210	220
2	220	242
3	230	266.20
4	240	292.82
5	250	322.10
6	260	354.31
7	270	389.74

Compare the rates of change. Jung's savings represent a linear function because the rate of change is constant. As the months increase by 1, the savings increase by 10. Miguel's savings represent a nonlinear function because as the months increase by 1, the total savings increases at a growing rate.

Jung's account can be represented by using slope-intercept form, $y = 10x + 200$, where x represents the number of months and y represents the amount in the savings account. Since there are 12 months in a year, the amount in Jung's account after one year can be found by replacing x with 12.

$y = 10(12) + 200$
    $= 120 + 200$
    $= 320$

After one year there will be $320 in Jung's account.

ANSWER:
a. False

b. False

c. True

Find each function value.

22. $f(5)$ if $f(x) = 3x + 4$

SOLUTION:

Substitute 5 for x into the function rule and evaluate.

$$f(x) = 3x + 4$$
$$f(5) = 3(5) + 4$$
$$f(5) = 15 + 4$$
$$f(5) = 19$$

ANSWER:

19

23. $f(–3)$ if $f(x) = 2x – 8$

SOLUTION:

Substitute –3 for x into the function rule and evaluate.

$$f(x) = 2x - 8$$
$$f(-3) = 2(-3) - 8$$
$$f(-3) = -6 - 8$$
$$f(-2) = -14$$

ANSWER:

–14

24. $f(7)$ if $f(x) = 9x – 24$

SOLUTION:

Substitute 7 for x into the function rule and evaluate.

$$f(x) = 9x - 24$$
$$f(7) = 9(7) - 24$$
$$f(7) = 63 - 24$$
$$f(7) = 39$$

ANSWER:

39

25. The table shows the average number of phone calls Riley makes per day.

Number of Days, d	Total Phone Calls, c
1	5
2	10
3	15
4	20

a. Write an equation to find the total number of phone calls made in any number of days. Describe the relationship in words.

b. Use the equation to determine how many phone calls Riley would make in 1 week.

SOLUTION:

a. Use the table to analyze the pattern.

Number of Days, d	Total Phone Calls, c
1	5
2	10
3	15
4	20

The pattern is 5 calls per day.

Let d represent the number of days and c represent the number of calls. The equation is $c = 5d$. Riley makes an average of 5 phones calls per day.

b. Write the equation. Replace d with 7. Multiply.

$c = 5d$

$c = 5(7)$

$c = 35$

Riley would make 35 phone calls in one week.

ANSWER:

a. $c = 5d$; Riley makes an average of 5 phone calls per day.

b. 35 phone calls