

Solving Unit Rate Problems

1 GETTING THE IDEA

A **rate** is a comparison, or ratio, of two quantities with different units. For example, a store sells 3 T-shirts for \$15. The comparison \$15 to 3 T-shirts is a rate. This rate can also be written as $\frac{\$15}{3 \text{ T-shirts}}$.

When a rate compares a quantity to one unit of another quantity, the rate is a **unit rate**.

The rate $\frac{\$15}{3 \text{ T-shirts}}$ is *not* a unit rate because the rate compares the cost to more than one T-shirt. In a unit rate, the second quantity (or denominator) should be 1 unit.

A speed limit is a unit rate. For example, the speed limit of 55 miles per hour compares the distance to 1 hour. A speed of 55 miles per hour is equivalent to traveling 55 miles in each 1-hour time span. This rate can also be written as $\frac{55 \text{ miles}}{1 \text{ hour}}$.

$\frac{\$15}{3 \text{ T-shirts}}$	$\frac{55 \text{ miles}}{1 \text{ hour}}$
not a unit rate	unit rate

Every rate can be written as a unit rate.

Example 1

A store sells 3 T-shirts for \$15. What is the cost per T-shirt?

Strategy Use a diagram to find the unit rate.

Step 1

Write the given rate as a fraction.

Because the question asks for the cost per T-shirt, the rate compares dollars to T-shirts.

$$\frac{\$15}{3 \text{ T-shirts}}$$

Step 2

Draw a diagram to represent the rate.

There are 15 dollars, so divide the top rectangle into 15 equal parts. There are 3 T-shirts, so divide the bottom rectangle into 3 equal parts.

dollars														
T- shirts														

Step 3

Use the diagram to find the unit rate.

The unit rate is the rate that compares the cost to one T-shirt.

dollars															
T- shirts															

There are five dollars for one T-shirt. So, the unit rate is $\frac{\$5}{1 \text{ T-shirt}}$, or \$5 per T-shirt.

Solution

The cost per T-shirt is \$5.

Example 2

Changing bikes 36 miles in 3 hours. How many miles per hour does she travel?

Strategy

Use an equation.

Step 1

Write a rate that compares distance to time.

$$\frac{\text{Distance}}{\text{time}} = \frac{36 \text{ miles}}{3 \text{ hours}}$$

Step 2

Write an equation.

The unit rate is the number of miles per 1 hour. The given rate and the unit rate are equivalent.

$$\frac{36 \text{ miles}}{3 \text{ hours}} = \frac{? \text{ miles}}{1 \text{ hour}}$$

Step 3

Use division to find the missing number.

Because $3 \text{ hours} \div 3 = 1 \text{ hour}$, write an equivalent rate by dividing 36 miles by 3:

$$\frac{36 \text{ miles} \div 3}{3 \text{ hours} \div 3} = \frac{12 \text{ miles}}{1 \text{ hour}}$$

Solution

Changing travels $\frac{12 \text{ miles}}{1 \text{ hour}}$, or 12 miles per hour.

Note that in both Example 1 and Example 2, the unit rate can be found by dividing the numerator by the denominator and simplifying the fraction. The unit rate for $\frac{\$15}{3 \text{ T-shirts}}$ is $15 \div 3 = 5$ dollars. The unit rate for $\frac{36 \text{ miles}}{3 \text{ hours}}$ is $36 \div 3 = 12$ miles per hour. This is because the quotient can be written as a fraction over a denominator of 1.

To find a unit rate, write the ratio as a fraction and divide the numerator by the denominator.

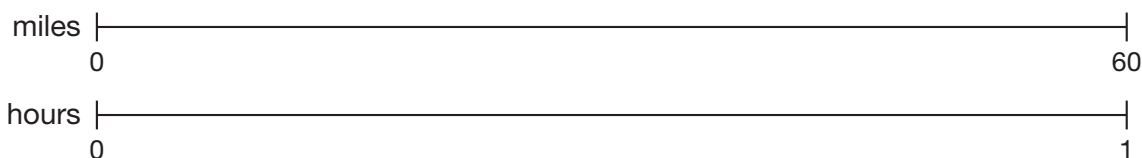
Example 3

Ramon drives at a rate of 60 miles per hour. He stops for lunch after driving for $\frac{1}{2}$ hour. How many miles did Ramon drive in $\frac{1}{2}$ hour?

Strategy Use a diagram to find the distance.

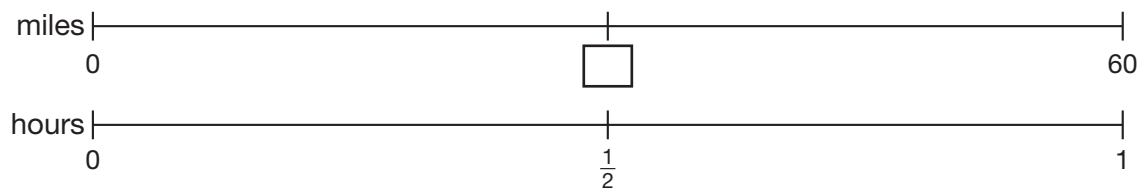
Step 1 Draw a diagram to show the given unit rate.

The given unit rate is $\frac{60 \text{ miles}}{1 \text{ hour}}$.



Step 2 Label the known and unknown quantities on the diagram.

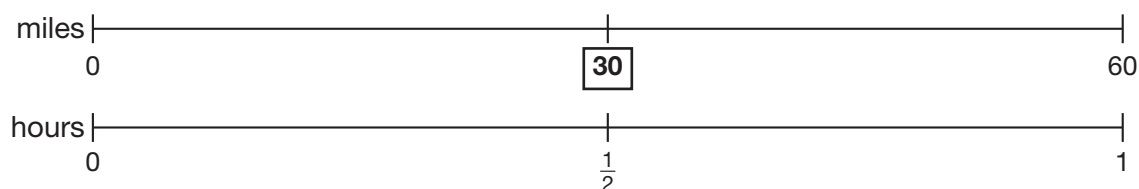
Ramon drove for $\frac{1}{2}$ hour. Show $\frac{1}{2}$ on the number line for hours.



Step 3 Use the diagram to find the unknown quantity.

The tick mark divides 60 into two equal parts.

Since $60 \div 2 = 30$, each part must represent 30.



Solution In $\frac{1}{2}$ hour, Ramon traveled 30 miles.

Example 4

Eva pays a company \$30 every 6 months to host her Web site.

- (a) How much does Eva pay per month?
- (b) How much does Eva pay for 9 months?

Strategy Use equations to find the equivalent rates.

Step 1

Write the given rate.

$$\frac{\text{Cost}}{\text{Time}} = \frac{\$30}{6 \text{ months}}$$

Step 2

Use division to find the cost per month.

Divide 30 by 6 to simplify the fraction and find the unit rate.

$$\frac{30}{6} = 30 \div 6 = 5$$

This is equal to $\frac{5}{1}$, or $\frac{\$5}{1 \text{ month}}$.

The unit rate is \$5 per month.

Step 3

Write an equation that can be used to answer part b.

Part b asks for the cost for 9 months. Use the unit rate to write an equation.

$$\frac{\$5}{1 \text{ month}} = \frac{?}{9 \text{ months}}$$

Step 4

Use multiplication to find the missing number.

Because $1 \text{ month} \times 9 = 9 \text{ months}$, write an equivalent rate by multiplying the numerator and denominator by 9:

$$\frac{\$5 \times 9}{1 \text{ month} \times 9} = \frac{\$45}{9 \text{ months}}$$

Solution Eva pays \$5 per month and \$45 for 9 months.

A water pump can pump 250 gallons from a pool in 5 minutes.

(a) What is the unit rate of the pump in gallons per minute?

(b) How long will it take to pump 450 gallons from the pool?

(a) The unit rate compares _____ to _____.

The given rate is _____ gallons in _____ minutes.

Write the given rate as a fraction.

$$\frac{\boxed{} \text{ gallons}}{\boxed{} \text{ minutes}}$$

Divide the numerator by the denominator to find the unit rate.

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

The unit rate is _____.

(b) Use the _____ rate to write an equation to solve the problem.

$$\frac{\boxed{} \text{ gallons}}{1 \text{ minute}} = \frac{450 \text{ gallons}}{? \text{ minutes}}$$

Multiply the numerator and denominator by _____ to find the equivalent fraction.

$$\frac{\boxed{} \times \boxed{} \text{ gallons}}{1 \times \boxed{} \text{ minutes}} = \frac{450 \text{ gallons}}{\boxed{} \text{ minutes}}$$

It takes _____ minutes to pump 450 gallons from the pool.

The unit rate of the pump is _____ gallons per minute. It takes _____ minutes to pump 450 gallons of water from the pool.

3 LESSON PRACTICE

- 1 Four muffins cost \$12. Complete the model to find the cost per muffin.

dollars	
muffins	

The cost per muffin is _____.

- 2 Select a rate that is a unit rate. Circle all that apply.

- A. $\frac{36 \text{ cookies}}{3 \text{ batches}}$
- B. 60 gallons every 4 hours
- C. \$40 per ticket
- D. $\frac{10 \text{ miles}}{10 \text{ hours}}$
- E. $\frac{1}{2}$ cup per serving
- F. 15 people for every row
- G. $\frac{12 \text{ roses}}{1 \text{ vase}}$

- 3 Draw a line from the rate to its unit rate.

- | | | |
|---|---|----------------------|
| A. $\frac{225 \text{ feet}}{5 \text{ minutes}}$ | • | • 18 feet per minute |
| B. $\frac{450 \text{ feet}}{9 \text{ minutes}}$ | • | • 30 feet per minute |
| C. $\frac{36 \text{ feet}}{2 \text{ minutes}}$ | • | • 45 feet per minute |
| D. $\frac{210 \text{ feet}}{7 \text{ minutes}}$ | • | • 50 feet per minute |

- 4 A line-painting truck is painting a line on the side of a roadway. The truck paints at a rate of 8 miles per hour. How many miles can be painted in $\frac{1}{4}$ hour?

_____ miles can be painted in $\frac{1}{4}$ hour.

- 5 Twelve boxes of pencils cost \$16. Which equation could you use to find the cost per box?

A. $\frac{12 \text{ boxes}}{\$16} = \frac{\boxed{} \text{ boxes}}{\$1}$

B. $\frac{\$16}{12 \text{ boxes}} = \frac{\$1}{\boxed{} \text{ boxes}}$

C. $\frac{12 \text{ boxes}}{\$16} = \frac{\$ \boxed{}}{1 \text{ box}}$

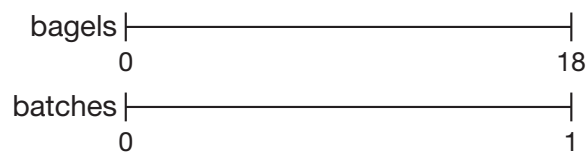
D. $\frac{\$16}{12 \text{ boxes}} = \frac{\$ \boxed{}}{1 \text{ box}}$

- 6 Cassie drives 165 miles in 3 hours. Circle the numbers that make the rates below equivalent to the rate 165 miles in 3 hours.

Cassie drives $\begin{array}{c} 55 \\ 60 \\ 65 \end{array}$ miles per hour.

Cassie drives $\begin{array}{c} 275 \\ 300 \\ 325 \end{array}$ miles in 5 hours.

- 7 Troy's recipe for bagels makes 18 bagels per batch. Troy makes $\frac{2}{3}$ batch of bagels. How many bagels does Troy make? Complete the model to solve the problem.



Troy makes _____ bagels.

8 Are the rates equivalent? Select True or False for each equation.

A. 35 feet for every 7 seconds = 7 feet for every second ☐ True ☐ False

B. $\frac{120 \text{ cans}}{5 \text{ cases}} = \frac{25 \text{ cans}}{1 \text{ case}}$ ☐ True ☐ False

C. \$12 per hour = \$48 for every 4 hours ☐ True ☐ False

D. $\frac{1 \text{ teacher}}{15 \text{ students}} = \frac{6 \text{ teachers}}{90 \text{ students}}$ ☐ True ☐ False

9 Takashi ran 200 yards in 40 seconds. Ana ran 150 yards in 25 seconds.

Part A

Can you tell who runs faster without rewriting the rates? Explain why or why not.

Part B

Who ran faster? Show your work.

- 10 Use numbers from the box to complete the table so that the rates are equivalent.

Dollars		90	180	
Hours	1	5		12

10
15
18
45
216
360

- 11 The speed limit is 50 miles per hour. Kyle is driving to a baseball game that starts in 2 hours. Kyle is 130 miles away from the baseball field. If Kyle drives at the speed limit, will he arrive in time? Show your work.

- 12 A new issue of Padma's favorite magazine comes out every month. If she buys the magazine at the bookstore, the cost is \$5 per month. If she subscribes, the cost is \$57 per year. Should Padma buy each issue separately or should she get a subscription? Show your work.