8.1 Matrices Enrichment

Def: Matrix - A matrix is an ordered array of numbers written in brackets

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & \boxed{9} \end{bmatrix}$$

→ row

→ element

column

Each number in the matrix is called an <u>element</u>, and matrices have <u>rows</u> and <u>columns</u>

Def: Order - The order of a matrix is the number of columns (m) by the number of Rows (n)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 1 \\ -1 & 5 & 6 \end{bmatrix}$$

2x2 → order

2x3

Def: Main Diagonal - Runs from upper left to lower right

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Ex: Given the following system

$$\begin{aligned} x - y + 3z &= 8 \\ 3x + y - 2z &= -2 \\ 2x + 4y + z &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & -1 & 3 \\ 3 & 1 & -2 \\ 2 & 4 & 1 \end{bmatrix} \rightarrow \text{coefficient matrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 8 \\ 3 & 1 & -2 & -2 \\ 2 & 4 & 1 & 0 \end{array}\right] \rightarrow \text{Augmented matrix}$$

To solve a system using matrices we must write the matrix in row echelon form. A matrix is in row-echelon form when all elements below the main diagonal are zeros.

$$\underbrace{\left[\begin{array}{ccc|c} 2 & 3 & 4 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 6 & 12 \end{array}\right]}_{\text{row echelon form}} \qquad \underbrace{\left[\begin{array}{ccc|c} 2 & 3 & 4 & 1 \\ 0 & 0 & 6 & 12 \\ 0 & 1 & 2 & 5 \end{array}\right]}_{\text{not row echelon form}}$$

To write a matrix in row-echelon we elementary-row operations.

R1: Row Scaling - Multiply the row by a non-zero scalar (constant)

R2: Row interchange - Switch any two Rows

R3: Row addition - Replace any row with the sum of that Row and any other row.

Def Gaussian Elimination Method - Method used to solve a system of equations by writing the coefficient portion of the augmented matrix in Row-echelon form using elementary row operations. We then use back substitution to solve.

→ named after Carl Friedrich Gauss (1777-1855) - a German mathematician.