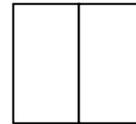
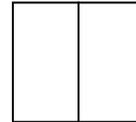


**Fencing Problems**

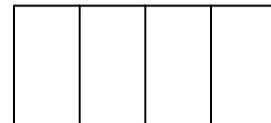
1. A farmer has 480 meters of fencing with which to build two animal pens with a common side as shown in the diagram. Find the dimensions of the field with the maximum area. What is the maximum area?



2. Max plans to build two side-by-side identical rectangular pens for his pigs that will enclose a total area of 216 square feet. What is the minimum length of fencing he will need? What are the dimensions of the total enclosure?



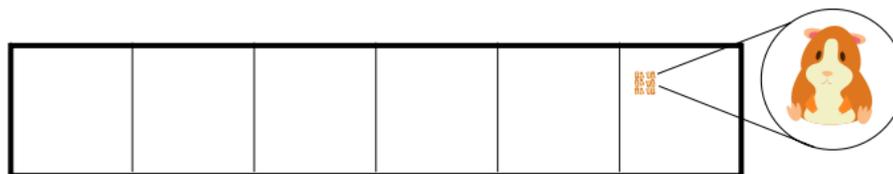
3. Build a rectangular pen with three parallel partitions using 500 feet of fencing. What overall dimensions will maximize the total area of the pen? What is the maximum area?



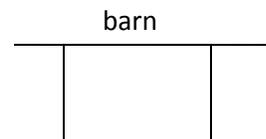
4. You are to fence a rectangular area. The fencing for the left and right sides costs \$20 per foot and the fencing for the front and back sides costs \$30 per foot. Find the dimensions of the rectangular area that result in the least cost, if the area inside the fencing is to be 3200 square feet. What is the cost? What are the dimensions?



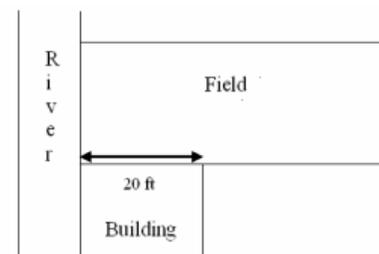
5. Your dream of becoming a hamster breeder has finally come true. You are constructing a set of rectangular pens in which to breed your furry friends. The overall area you are working with is 60 square feet, and you want to divide the area up into six pens of equal size as shown below. The cost of the outside fencing is \$10 a foot. The interior fencing costs \$5 a foot. You wish to minimize the cost of the fencing. Find the minimum cost and the overall dimensions of the enclosure.



6. A rectangular lettuce patch, 480 square feet in area, is to be fenced off against rabbits. Find the least amount of fencing required if one side of the land is already protected by a barn. What are the dimensions?

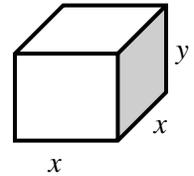


7. A rectangular field as shown is to be bounded by a fence. Find the dimensions of the field with maximum area that can be enclosed with 1000 feet of fencing. You can assume that fencing is not required along the river and the building. What is the max area?



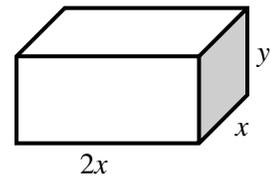
## Boxes (Rectangular Prisms)

1. An open-top rectangular box with square base is to be made from 48 square feet of material. What dimensions will result in a box with the largest possible volume? What is the volume?



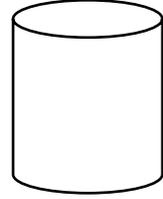
2. An open-top rectangular box with square base is to be made from 1200 square cm of material. What dimensions will result in a box with the largest possible volume? What is the volume?
3. A closed-top rectangular container with a square base is to have a volume  $300 \text{ in}^3$ . The material for the top and bottom of the container will cost  $\$2$  per  $\text{in}^2$ , and the material for the sides will cost  $\$6$  per  $\text{in}^2$ . Find the dimensions of the container of least cost. What is that cost?

4. An open-top box will be constructed with material costing  $\$7$  per square meter for the sides and  $\$13$  per square meter for the bottom. The dimensions of the bottom are to have its length equal to twice its width (see diagram). Find the dimensions of the box of largest volume than can be built with at most  $\$300$  of materials. What is the volume?

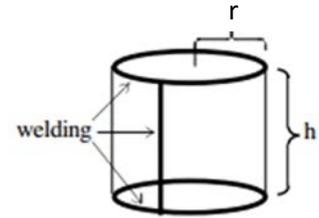


5. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost  $\$10/\text{ft}^2$  and the material used to build the sides cost  $\$6/\text{ft}^2$ . If the box must have a volume of  $50\text{ft}^3$  determine the dimensions that will minimize the cost to build the box.
6. A pool with a square bottom is to have a volume of 2000 cubic feet. The owners plan to use a fancy tile to complete the pool. The sides of the pool will cost  $\$80$  per square foot and the bottom of the pool will cost  $\$40$  per square foot. Find the pool dimensions that will minimize the cost of construction.
7. Find the dimensions of the least expensive rectangular box which is three times as long as it is wide and which holds 100 cubic centimeters of water. The material for the bottom costs  $7\text{¢}$  per  $\text{cm}^2$ , the sides cost  $5\text{¢}$  per  $\text{cm}^2$  and the top cost  $2\text{¢}$  per  $\text{cm}^2$ .

**Cylinders:** Formulas:  $V = \pi r^2 h$ ; Surface Area = 2 circles + lateral area =  $2\pi r^2 + 2\pi rh$



1. A cylindrical can with a closed top is made to contain  $150 \text{ cm}^3$  of liquid. Find the radius and height that will minimize the surface area of the metal to make the can. What is the minimum surface area?
2. A container in the shape of a right circular cylinder with no top has surface area of  $42\pi \text{ ft}^2$ . What height and base radius will maximize the volume of the cylinder? What is the maximum volume?
3. A closed cylindrical container is to have a volume of  $300 \pi \text{ in}^3$ . The material for the top and bottom of the container will cost  $\$2$  per  $\text{in}^2$ , and the material for the side will cost  $\$6$  per  $\text{in}^2$ . Find the dimensions of the container of least cost. What is that cost?
4. A can company wishes to produce a cylindrical container with the capacity of 1250 cubic centimeters. The top and bottom of the container must be made of material that costs  $\$0.05$  per square centimeter, while the material for the side is  $\$0.03$  per square centimeter. Find the dimensions that will minimize the cost of the container. What is the cost?

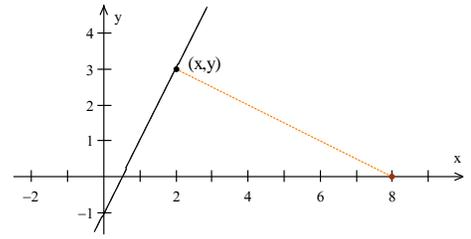


5. Find the height and radius of the least expensive closed cylinder which has a volume of 1000 cubic inches. Assume that the materials are free, but that it cost  $80\text{¢}$  per inch to weld the top and bottom discs onto the cylinder and to weld the seam up the side of the cylinder.
6. The surface of a can is 500 square centimeters. Find the dimensions of the can with the greatest volume. What is that volume?
7. Find the maximum volume of a cylinder whose radius and height add up to 24 inches. What are the length of the radius and the height of this cylinder?

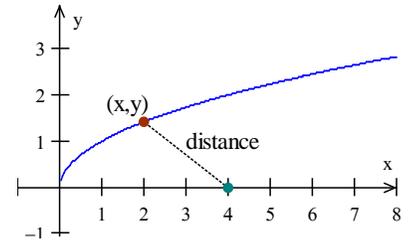
**Distance formula:**  $(x_1, y_1)$   $(x_2, y_2)$   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  (Note this is just a fancy Pythagorean Theorem)

**Distances:**

1. Find the point on  $y = 2x - 1$  that is closest to  $(8,0)$ . What is the distance between the two points?



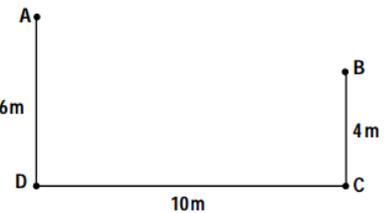
2. Find the point on  $y = \sqrt{x}$  that is closest to  $(4,0)$ . What is the distance between the two points?



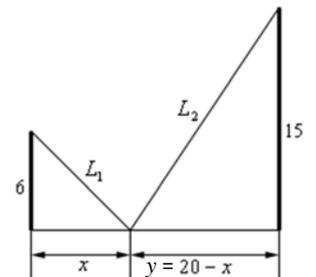
3. Find the point on  $y = 4 - x^2$  that is closest to  $(5, 2)$ . What is the distance between the two points?

4. Find the point on  $y = x^3$  that is closest to  $(4, -1)$ . What is the distance between the two points?

5. Two poles, one 6 meters tall and the other 4 meters tall, are 10 meters apart. A length of wire is attached to the top of each pole and it is also staked to the ground somewhere between the two poles. How far from point C should the wire be staked so that a minimum amount of wire is used? What is the minimum amount of wire?

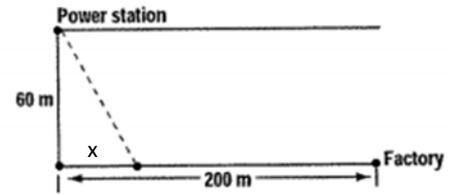


6. Two poles, one 6 meters tall and the other 15 meters tall, are 20 meters apart. A length of wire is attached to the top of each pole and it is also staked to the ground somewhere between the two poles. Where should the wire be staked so that a minimum amount of wire is used? What is the minimum amount of wire?



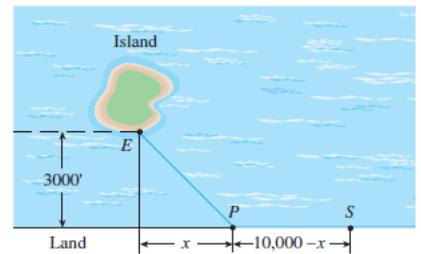
## Minimizing Costs of Cables (and similar problems involving rates)

1. A power station and a factory are on opposite sides of a river 60m wide as shown. A power line must be run from the station to the factory. It costs \$25 per meter to run the cable in the river and \$20 per meter on land. How should the cable be placed in order to minimize the total cost?

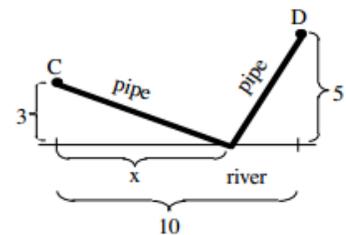


2. You want to run an underground power cable from a power station on one side of a river to a house on the other side. The house is 5 miles downstream from the station, and the river has a constant width of 1 mile. It costs \$1000 per mile to lay cable underground, and \$3000 per mile to lay cable under water. How should you place the cable to minimize the total cost, and what will the minimum cost be?

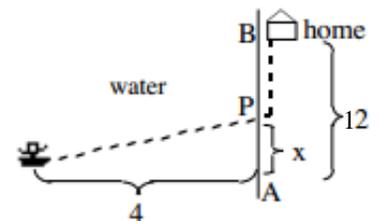
3. Find the cheapest possible cost to run a power cable from a power station (on the coast, S) to a marine biology station (on an island). The cost of the cable across land is \$11.50 per linear foot and the cost of the cable under water is \$21.50 per linear foot. The coast is straight and optimal cost will result in laying a combination of straight cable across some of the land and across some path through the water. See picture for additional information.



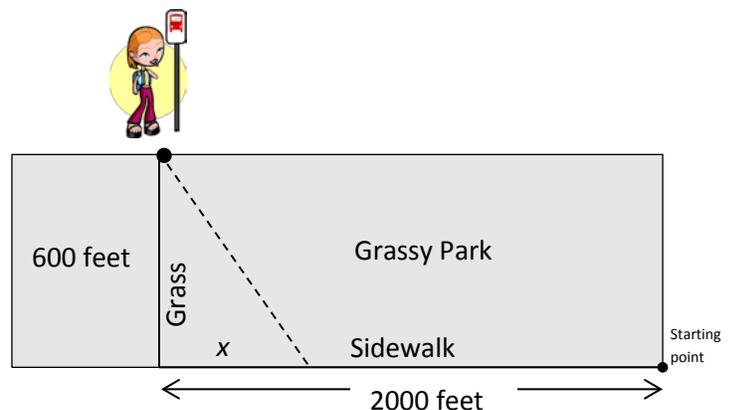
4. You have been asked to determine where a water works should be built along a river between Chesterville and Denton to minimize the total cost of the project. The pipe to Chesterville costs \$3000 per mile and the pipe to Denton costs \$7000 per mile. Find the length of each pipe so that the total cost is a minimum. What is the cost?



5. You need to reach home as quickly as possible, but you are in a rowboat 4 miles from shore and your home is 12 miles up the coast (see drawing). If you can row at 3 miles per hour and walk at 5 miles per hour, toward which point on the shore should you row (in other words, find the length of  $x$  such that your travel time is a minimum)? What is the minimum time?



6. Elaine walks to the same bus stop every day and frequently just misses her bus. She wants to know what path will get her to the bus stop in the shortest time. The bus stop is across a grassy park, 2000 feet West and 600 feet north of her starting position. Elaine can walk west along the edge of the park (on the sidewalk) at a speed of 6 feet per second. She can also travel through the grass in the park, but only at a rate of 4 feet per sec. What path will get her to the stop the fastest?



Answers:

**Fencing:**

1. overall dimensions: 80m x 120m; Maximum Area =  $9600\text{m}^2$
2. overall dimensions: 12 feet x 18 feet; Minimum Fencing: 72 feet
3. overall dimensions: 50 feet x 125 feet; Maximum Area =  $6,250\text{ft}^2$
4. overall dimensions: 46.188 x 69.282; Minimum Cost = \$5542.56
5. overall dimensions: 5.164 feet x 11.619; Minimum Cost = \$464.76
6. dimensions: 15.492 x 30.984; Minimum Fencing: 61.968 ft
7. dimensions: 255 feet x 510 feet; Maximum Area =  $130,050\text{ft}^2$

**Boxes:**

1.  $x = 4\text{ ft.}$  and  $y = 2\text{ ft.}$ , Volume =  $32\text{ft}^3$
2.  $V = 4000\text{ cm}^3$ ; box dimensions are 20 x 20 x 10 cm.
3. The cost is minimized when the dimensions are  $9.655 \times 9.655 \times 3.218$ . The cost is \$1118.60.
4. Dimensions: 1.96 x 3.92 x 2.43 m; Maximum Volume =  $18.68\text{ m}^3$ .
5. Dimensions: 1.882 x 5.646 x 4.705 ft; Minimum Cost = \$637.60
6. Dimensions are 20 x 20 x 5 feet; Minimum Cost is \$48,000.
7. Dimensions are 2.912 x 8.736 x 3.931 cm, Minimum Cost = \$6.87

**Cylinders:**

1.  $r = 2.879\text{ cm}$ ,  $h = 5.759\text{ cm}$ , Minimum Surface Area =  $156.282\text{ cm}^2$
2.  $r = 3.742\text{ ft}$ ,  $h = 3.742\text{ ft}$ , Maximum Volume =  $164.567\text{ ft}^3$
3.  $r = 7.663\text{ in}$ ,  $h = 5.109\text{ in}$ , Minimum Cost = \$2213.81
4.  $r = 4.924\text{ cm}$ ,  $h = 16.412\text{ cm}$ , Minimum Cost = \$22.85
5.  $r = 3.700\text{ in}$ ,  $h = 23.249\text{ in}$ , Minimum Cost = \$55.78
6.  $r = 5.150\text{ cm}$ ,  $h = 10.301\text{ cm}$ , Maximum Volume =  $858.387\text{ cm}^3$
7.  $r = 16\text{ in}$ ,  $h = 8\text{ in}$ , Maximum Volume =  $6433.98\text{ in}^3$

**Distances:**

1. point: (2, 3) Minimum Distance: 6.708
2. point: (3.5, 1.871) Minimum Distance: 1.936
3. point: (1.719, 1.046) Minimum Distance: 3.417
4. Point: (0.824, 0.559) Minimum Distance: 3.538
5. Point C to Stake: 4 meters; Minimum Wire Length: 14.142 m
6.  $x = 5.714\text{ m}$ ; Minimum Wire Length: 29 m

**Cables & others...:**

1.  $x = 80\text{ m}$ ; Minimum Cost = \$4900
2.  $x = 0.353\text{ m}$ ; Minimum Cost = \$7828.42
3.  $x = 1899.162\text{ ft}$ ; Minimum Cost = \$169,497.71
4.  $x = 7.817\text{ miles}$ ; Minimum Cost = \$63,309.14
5.  $x = 3\text{ miles}$ ; Shortest Time = 3.47 hours
6.  $x = 537\text{ ft}$ ; Stay on the sidewalk for 1463 ft. Shortest time: 445 seconds (7.4 minutes)